Tentamen: Subatomaire fysica

Thursday 25 february 1999

1. (4 pt.) Parity conservation

a) Prove that parity conservation forbits a static electric dipole moment in the ground state of a stable nucleus. ($\vec{D} \sim \vec{r}$)

$$\langle \vec{D} \rangle \approx \int d^3\vec{r}\, \psi^*(\vec{r})\, \vec{r}\, \psi(\vec{r}) = -\int d^3\vec{r}\, \psi^*(-\vec{r})\, \vec{r}\, \psi(-\vec{r}) = -\int d^3\vec{r}\, \psi^*(\vec{r})\, \vec{r}\, \psi(\vec{r}) = -\langle \vec{D} \rangle$$

Parity operation ($\vec{r} \to -\vec{r}$)

The ground state of a nucleus has a definite parity so: $\psi(-\vec{r}) = \pm \psi(\vec{r})$

Thus $\psi^*(-\vec{r})\psi(-\vec{r}) = (\pm 1)^2 \psi^*(\vec{r})\psi(\vec{r}) = \psi^*(\vec{r})\psi(\vec{r})$

$\Rightarrow$ $\langle \vec{D} \rangle = -\langle \vec{D} \rangle$ implies $\langle \vec{D} \rangle = 0$, so there is no static dipole moment.

b) Prove that a non-zero expectation value of the helicity operator λ violates parity conservation. ($\lambda = \vec{\sigma} \cdot \vec{p}/|\vec{p}|$)

$$\langle \lambda \rangle = \int d^3\vec{r}\, \psi^*(\vec{r})\, \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\, \psi(\vec{r}) = -\int d^3\vec{r}\, \psi^*(-\vec{r})\, \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\, \psi(-\vec{r}) =$$

parity op: $\vec{r} \to -\vec{r}$, $\vec{p} \to -\vec{p}$, $\vec{\sigma} \to \vec{\sigma}$

$$= -\int d^3\vec{r}\, \psi^*(\vec{r})\, \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\, \psi(\vec{r}) = -\langle \lambda \rangle$$

If we have parity conservation, the system must be in a definite eigenstate ψ of the parity operator. ~~Thus~~ So we have the same ~~sit~~ situation as in a//.

We conclude: $\langle \lambda \rangle = 0$ if there is parity conservation, if $\langle \lambda \rangle \neq 0$ parity conservation is violated.

2. (14 pt.) Consider a nuclear system built from 2 nucleons.

a) Write down the possible isospin wavefunctions for the coupling of 2 nucleons to the di-nucleon system. (use the notation $|I, I_3>$ for the isospin I and 3-component I_3)

We have $2 \otimes 2$ coupling in $SU(2)$: $2 \otimes 2 = 3 \oplus 1$, so we get a $I=1$ triplet and a $I=0$ singlet:

$I = 1$:

- $I_3 = +1$: $|1,1\rangle = |\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$
- $I_3 = 0$: $|1,0\rangle = \frac{1}{\sqrt{2}}\left\{|\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle + |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle\right\}$
- $I_3 = -1$: $|1,-1\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$I = 0$. $I_3 = 0$: $|0,0\rangle = \frac{1}{\sqrt{2}}\left\{|\tfrac{1}{2},\tfrac{1}{2}\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle - |\tfrac{1}{2},-\tfrac{1}{2}\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle\right\}$

Note: * nucleons are members of a $I=\frac{1}{2}$ doublet ($I=\frac{1}{2}$, $I_3 = \pm\frac{1}{2}$)

* The coefficients in the wavefunctions are obtained from Clebsch-Gordan table (i).

2 (b) What is the possible spin (J) assignment for the isospin-triplet and -singlet states, respectively?

$\Psi = \varphi(\text{space})\, X(\text{spin})\, I(\text{isospin})$ ← anti symmetric! (we have 2 fermions!)

Triplet: I = symmetric so : either φ(space) = even and X(spin) = antisymmetric or
φ(space) = odd , X(spin) = symmetric. } $\underline{J=0}$

Considering the ground state (L=0) we have φ = even, so X = antisymm ⇒ $\underline{S=0}$

Singlet: I = antisymm so either φ(space) = even and X(spin) = even or
φ(space) = odd and X(spin) = odd. } $\underline{J=1}$

Grand state: (L=0) we have φ = even and thus X = even ⇒ $\underline{S=1}$

2 c) The ground-state of the bound di-nucleon system, the deuteron, is the $^3S_1$ state, i.e. spin J=1 and orbital angular momentum l=0, while the $^1S_0$ state is unbound. Discuss the binding of the di-proton and the di-neutron on basis of the isospin-independence of the nuclear force.

The ~~total~~ nuclear force only depends on the total isospin I, $\underline{\text{not}}$ on I_3.
As showed in b), the $^3S_1$ state corresponds to I = 0 (isospin singlet)
The $^1S_0$ state corresponds to I = 1 (triplet). Because di-proton and di-neutron are part of the I = 1 triplet and the nuclear force is not enough to bind $|1,0\rangle_{\text{isospin}}$ it will (because of its independence of I_3) $\underline{\text{not}}$ be able to bind $|1,1\rangle$ (di-proton) and $|1,-1\rangle$ (di-neutron) as well.

1.5 (d) Determine the magnetic dipole moment μ/μ_N of the deuteron for a pure $^3S_1$ state. ($g_l = 1$; g_s = 5.586 for the proton; $g_l = 0$; g_s = -3.826 for the neutron)
The calculated value is about 2.5% larger than the observed value. What could be the reason?

Because we have $^3S_1$, there is no magnetic moment due to orbital motion (L=0). The spins are parallel, so we can just $\underline{\text{add}}$ the two seperate μ's to obtain the correct value:

$$\frac{\mu}{\mu_N} = \frac{\mu_{\text{proton}}}{\mu_N} + \frac{\mu_{\text{neutron}}}{\mu_N} = \frac{1}{2} g_{s,\text{proton}} + \frac{1}{2} g_{s,\text{neutron}} = \frac{1}{2}(5,586 - 3,826) = \frac{1}{2} 1,76.$$

The ~~nuclear~~ gluons (which can form virtual quark-antiquark pairs) contribute to the magnetic moment as well. { We could also have mixing between the pure $^3S_1$ state and ~~$^1S_0$~~ $^3D_1$)

2 e) What value of the electric quadrupole moment would you expect for the deuteron in a pure S-state?

Zero, a pure S-state is spherically symmetric, so there is (by definition) no quadrupole-moment.

1 f) The measured value of the electric quadrupole moment of the deuteron is Q=0.00288 b. Is this "small" or "large" and how can you explain this value qualitatively?

This is quite large compared to the expected value for a pure S-state.

This can be explained: there is $\underline{\text{mixing}}$ between the S-state and the energetically higher $\underline{\textbf{D}}$-state. The spherical symmetry is lost and so there is a quadrupole moment. (same parity!)

Parity conservation : $J^P = 1^+$: $l=0, {}^3S_1$

$l=2, {}^3D_1$

0

g) Which symmetry principle determines the allowed admixture to the deuteron ground state and which state is the most likely admixture?

~~The P-state is most likely; of all orbital angular momenta states is has the lowest energy (excluding S!), so it is the most likely.~~

~~Symmetry principle: we must have a J = 1 state, this we can get by S=1 and L=0, but also by S = 0 and L = 1 (our orbital P-state!)~~

~~The principle is thus SU(2) symmetry: coupling of angular momenta.~~

3 3. (4 pt.) Spin dependent potential.

Low-energy neutron-proton scattering data can be described approximately by representing the internucleon potential by an attractive square-well: range a=2fm, depth =35 MeV in the ${}^3S_1$ state and depth = 15 MeV in the ${}^1S_0$ state. This potential can be expressed as follows

$V(r) = A + B\, \vec{s_1} \cdot \vec{s_2}$ for $r \le a$,

$V(r) = 0$ for $r > a$,

where $\vec{s_1}$ and $\vec{s_2}$ are the nucleon spins.

Determine the values A (in MeV) and B (in the appropriate unit.)

$S(S+1)$

$$\vec{S}^2 = (\vec{s}_1+\vec{s}_2)^2 = \vec{s}_1^{\,2} + \vec{s}_2^{\,2} + 2\,\vec{s}_1\cdot\vec{s}_2 \Rightarrow \vec{s}_1\cdot\vec{s}_2 = \tfrac{1}{2}\vec{S}^2 - \tfrac{1}{2}\vec{s}_1^{\,2} - \tfrac{1}{2}\vec{s}_2^{\,2} = \tfrac{1}{2}\vec{S}^2 - \tfrac{1}{4} \;!$$

$s_1 = s_2 = \frac{1}{2}$

For ${}^3S_1$: $\vec{s}_1\cdot\vec{s}_2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow V(r) = 35 = A + \frac{1}{4}B$

${}^1S_0$: $\vec{s}_1\cdot\vec{s}_2 = 0 - \frac{1}{4} = -\frac{3}{4} \Rightarrow V(r) = 15 = A - \frac{1}{4}B$

$2A = 50 \Rightarrow A = 25$ (MeV)

$B = (35-A)4 = 40 \left(\mathrm{MeV}/\hbar^2\right)$

There should be an additional minus sign, because we have a well.

So: $V(r) = -\cancel{25}\,30\,(\mathrm{MeV}) - \cancel{40}\,20 \left(\frac{\mathrm{MeV}}{\hbar^2}\right) \underbrace{\vec{s}_1\cdot\vec{s}_2}_{\text{these are in } \hbar}$ so $\vec{s}_1\cdot\vec{s}_2 \propto \hbar^2$. So to get V(r) as an energy B must have $\frac{\mathrm{MeV}}{\hbar^2}$.

3 4. (6 pt.) Spin-orbit coupling.

a) Express $\vec{l}\cdot\vec{s}$ in terms of j, l and s. and show that the energy separation of a nuclear spin-orbit doublet is proportional to $2l+1$.

$$\vec{J} = (\vec{L}+\vec{S}) \Rightarrow \vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L}\cdot\vec{S} \Leftrightarrow \vec{L}\cdot\vec{S} = \tfrac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$$

acting on a nuclear state $|j, l, s\rangle$

$$= \tfrac{1}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$$

take $\hbar^2 = 1$.

A spin-orbit doublet consists of same L and S and $J = L \pm \frac{1}{2}$ ($S=\frac{1}{2}$!)

So $\Delta E \propto \Delta(\vec{l}\cdot\vec{s})$ = $\frac{1}{2}\left[(l+\frac{1}{2})(l+\frac{3}{2}) - l(l+1) - \frac{1}{2}(\frac{1}{2}+1)\right] - \frac{1}{2}\left[(l-\frac{1}{2})(l+\frac{1}{2}) - l(l+1) - \frac{1}{2}(\frac{1}{2}+1)\right]$

spin orbit coupling

$$= \tfrac{1}{2}\left[\cancel{l^2} + \tfrac{1}{2}l + \tfrac{3}{2}l + \tfrac{3}{4} - \cancel{l^2} - l - \tfrac{1}{4} - \tfrac{1}{2}\right] - \tfrac{1}{2}\left[\cancel{l^2} - \cancel{\tfrac{1}{2}l} + \cancel{\tfrac{1}{2}l} - \tfrac{1}{4} - \cancel{l^2} - l - \tfrac{1}{4} - \tfrac{1}{2}\right]$$

$$= \tfrac{1}{2}l - \tfrac{1}{2}[-l-1] = l + \tfrac{1}{2} = \tfrac{1}{2}(2l+1) \propto (2l+1)$$

qed.

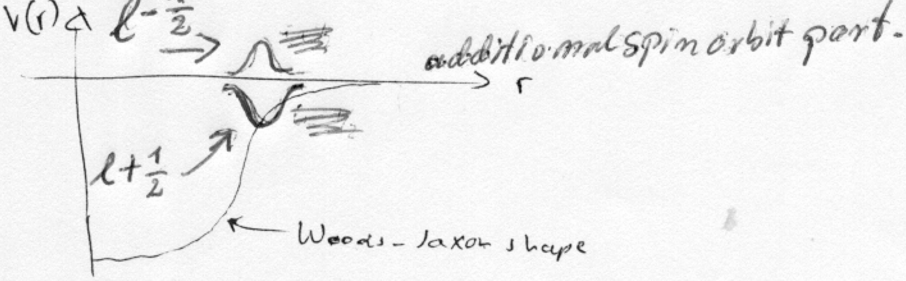
2 b) In the shell model the radial dependence of the nuclear density is assumed to be of the Woods-Saxon shape.
Obtain an expression for the spin-orbit potential and sketch the radial dependence for $j=l\pm\frac{1}{2}$.

Woods-Saxon: $\rho(r) = \dfrac{+\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}$ (density $\rho(r)$)

positive for $j = l + 1/2$
negative $j = l - 1/2$

$$\text{Spin-orbit} \propto \frac{d\rho(r)}{dr}\,\vec{l}\cdot\vec{s} = \frac{-V_0}{\left(1+\exp\left(\frac{r-R}{a}\right)\right)^2}\exp\left(\frac{r-R}{a}\right)\cdot\frac{1}{R}\,(\vec{l}\cdot\vec{j}) = \frac{V_0}{R}\,\frac{\exp\left(\frac{r-R}{a}\right)}{\left[1+\exp\left(\frac{r-R}{a}\right)\right]^2}\,\vec{l}\cdot\vec{j}.$$

sketch:



5. (6 pt.) Single-particle shell model.
3 The single-particle levels in a Woods-Saxon potential with spin-orbit coupling are given in appendix A.
a) Argue, why the ground state and first excited states of $^{45}_{21}$Sc have spin^{Parity} $\frac{7}{2}^-$, $\frac{3}{2}^+$, and $\frac{3}{2}^-$, respectively.

There is an odd number of protons, even number of neutrons (even number, DOES NOT CONTRIBUTE TO J^P). So there is one unpaired proton, which contributes to J^P. This proton is number 21. So in its ground state it is the only ~~particle~~ proton filling $1f_{7/2}$ ($1d_{3/2}$ is filled at 20 protons, so there's one left for $1f_{7/2}$). Parity of f-state is $(-1)^l = (-1)^3 = -1$ so $J^P = 7/2^-$. The unpaired proton can be excited in its 1st and 2nd excited state, $2p_{3/2}$ and $1f_{5/2}$. The latter is energitically larger than creating a hole in $1d_{3/2}$ (the proton leaving $1d_{3/2}$ can pair with the number-21-proton in $1f_{7/2}$). So we get: ground state $7/2^-$ caused by unpaired proton in $1f_{7/2}$. 1st excited state $3/2^-$ by unpaired proton in $2p_{3/2}$, 2nd excited state $3/2^+$ by hole in $1d_{3/2}$ and two paired protons in $1f_{7/2}$.

b) The magnetic moment for a nucleus with spin J is given by
3 $\mu_J = J\, g_J\, \mu_N$ with
$g_J = g_l \pm \frac{g_s - g_l}{2l+1}$ and
$g_l = 1$; $g_s = 5.586$ for the proton;
$g_l = 0$; $g_s = -3.826$ for the neutron.
Calculate the ground-state magnetic moment of $^{45}_{21}$Sc (in units of μ_N).

Ground state: $1f_{7/2}$, caused by one proton.

$$g_J = g_l \pm \frac{g_s - g_l}{2l+1} = 1 + \frac{5,586-1}{2\cdot 3+1} = 1,655 \Rightarrow \mu_J = J\, g_J\, \mu_N$$

+: spin and l are aligned ($j = l + \frac{1}{2}$)

$$= \frac{7}{2}\, g_J\, \mu_N = 5,793\ \mu_N.$$

2

6. (6 pt.) Shell filling.

Since the nuclear force is short-range attractive the lowest-energy state between two nucleons is achieved if their wavefunctions overlap maximally, i.e. if their angular momenta are oriented antiparallel. This pairing force increases strongly with the value of the angular momentum l. In this light, interpret the configuration of the following nuclei using the SPSM results in appendix A:

a) the ground states of

${}^{199}_{80}\mathrm{Hg}_{119}$, ${}^{203}_{81}\mathrm{Tl}_{122}$, ${}^{207}_{82}\mathrm{Pb}_{125}$, which have spin^{Parity} values $J^P = \frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$, respectively;

The neutrons and protons can be considered to enter levels paired, ~~one~~ with opposite m_J-values. First the levels of highest $|m_J|$ are filled, according to the pairing-force-principle, then the lower $|m_J|$, until the level is filled. Then it starts over again, until all ~~pr~~ protons and neutrons have been used. The neutron/proton left then determines J^P.

For ${}^{199}_{80}Hg_{119}$: one unpaired neutron ~~enters~~ is expected in $2f_{5/2} \rightarrow J^P = \frac{5}{2}^-$ but goes to $3p 1/2$ ($\frac{1}{2}^-$!!) because of lower l !

For ${}^{203}_{81}Tl_{122}$ one unpaired proton enters $3s_{1/2} \rightarrow J^P = \frac{1}{2}^+$

For ${}^{207}_{82}Pb_{125}$ there is a neutron hole in $3p_{1/2}$, so $J^P = 1/2^-$

b) the low-lying levels of ${}^{13}_{6}\mathrm{C}$ which are in the notation J^P(excitation energy): $\frac{1}{2}^-$(0 MeV, groundstate); $\frac{1}{2}^+$(3.09 MeV); $\frac{3}{2}^-$(3.68 MeV); $\frac{5}{2}^+$(3.85 MeV).

Ground-state: 2 protons in $1s_{1/2}$, 4 " in $1p_{3/2}$; 2 neutrons in $1s_{1/2}$, 4 " in $1p_{3/2}$; 1 neutron in $1p_{1/2}$ $\rightarrow$ $J^P = \frac{1}{2}^-$

Now it's energetically most favourable to excite a proton from $1p_{3/2}$ to $1p_{1/2}$ $\rightarrow$ we get a hole in $1p_{3/2}$: $(\frac{3}{2})^-$ (3,68 MeV)

~~The $1p_{1/2}$ level is now full,~~

We could also excite the proton from $1p_{1/2}$ to $1d_{5/2}$: $(\frac{5}{2})^+$ (3,85 MeV)

How to explain $\frac{1}{2}^+$ (3,09 MeV) I don't know. hole in $1S 1/2$, because of lower l value!

2

8

7. (8 pt.) γ transitions.

For the following γ transitions, name the permitted multipoles and indicate which multipole might be the most intense in the emitted radiation:

a) $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$

$|J_i - J_f| \leq L \leq J_i + J_f$: $1 \leq L \leq 8$

Parity for Electric multipoles: $(-1)^L$

" " magnetic " $(-1)^{L+1}$

parity of multipole must be +1 by parity conservation, so allowed multipoles are: M1, E2, M3, E4, M5, E6, M7, E8.

Lowest L, highest probability so: M1 might be most intense

b) $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$

$|J_i - J_f| \leq L \leq |J_i + J_f| \iff 3 \leq L \leq 4$

parity of multipole again +1.

So: M3, E4

~~E~~ most intense.

c) $1^- \to 2^+$

$|J_i - J_f| \leq L \leq |J_i + J_f| \iff 1 \leq L \leq 3$. Parity of multipole $\underline{\underline{-1}}$.

So $E1, M2, E3$.
↑ most intense.

d) $4^+ \to 2^+$

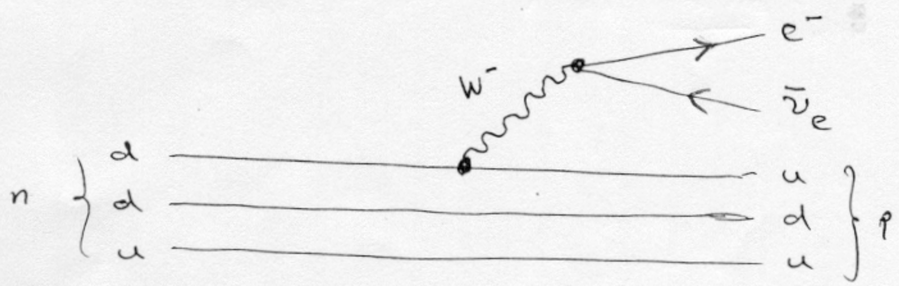
$|J_i - J_f| \leq L \leq |J_i + J_f| \iff 2 \leq L \leq 6$ Parity $+1$

So: $E2, M3, E4, M5, E6$
↑ most intense.

8. (6 pt.) β decay.

a) Draw the Feynman diagram for the β decay of the neutron on the quark level.

3



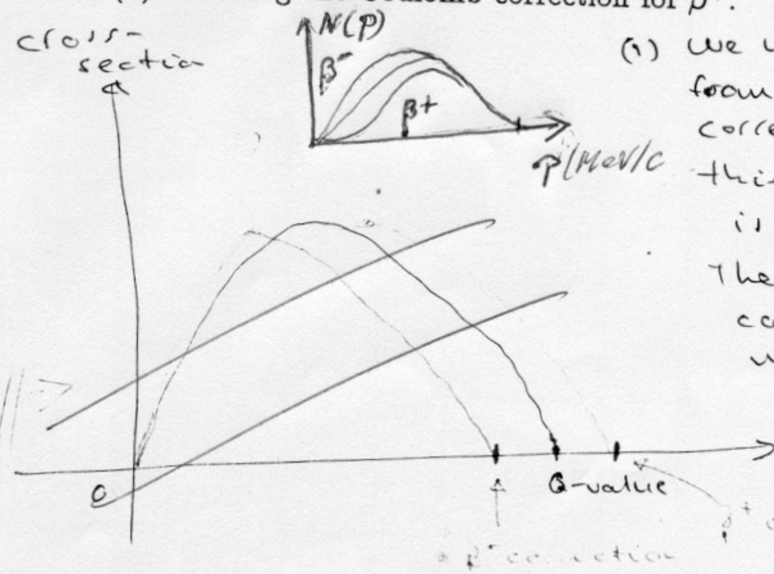
b) Sketch and motivate the general behaviour of the momentum spectrum of β particles for 3 cases:

2

(1) neglecting the correction due to the Coulomb field of the final nucleus;

(2) including the Coulomb correction for β^-;

(3) including the Coulomb correction for β^+.



(1) We have a continuous spectrum from 0 to the momentum corresponding to the Q-value, this is because the neutrino is given momentum too! The Q-value is the energy coming free due to mass-differences, this is divided between e^- and $\bar{\nu}_e$

(2) β^-: the positive core decellerates the β^- → the momentum (average and maximum) becomes smaller

(3) β^+: the core accellerates the β^+ ⇒ Larger Momentum.

4

9. (4 pt.) **Parity conservation.**
The pseudoscalar $\eta(547)$ meson is observed to decay to 3-pion final states. Explain, why the decay $\eta \to \pi^+\pi^-$ and $\eta \to \pi^0\pi^0$ have never been observed.

This ~~are~~ strong decays (no changes in quark flavour)
In strong decays parity is conserved!

Initial state $J^P = 0^-$. Final states consists of two pseudoscalar mesons (both: $J^P = 0^-$) Because of conservation of angular momentum the pions must be in a relative S-state. (both $\pi^+\pi^-$ and $\pi^0\pi^0$ have intrinsic spin 0).
Parity of final state is equal for both decays, namely: $\underbrace{(-1)^L}_{L=0} \cdot \underbrace{(-1)\cdot(-1)}_{\text{intrinsic parities of mesons}} = +1$
Parity of initial state is -1. Parity-violation ☹
Decay impossible.

4

10. (4 pt.) **Baryon resonances.**
In comparison to π^+p scattering, the cross section for π^-p scattering shows additional structure. How can you explain this and what is the relative weight of the contributing amplitudes? (use appendix C)

π^+, π^- part of isospin 1 triplet. p part of isospin $\frac{1}{2}$ doublet.

$$|\pi^+ p\rangle = |1,1\rangle |\tfrac{1}{2},\tfrac{1}{2}\rangle \underset{\text{appendix C}}{=} |\tfrac{3}{2},\tfrac{3}{2}\rangle$$

$$|\pi^- p\rangle = |1,-1\rangle |\tfrac{1}{2},\tfrac{1}{2}\rangle \overset{\downarrow}{=} \sqrt{\tfrac{1}{3}}\, |\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\, |\tfrac{1}{2},-\tfrac{1}{2}\rangle .$$

The strong interaction is dependent on total isospin.
For the $|\pi^+ p\rangle$ ~~case~~ scattering there is only a $I = \frac{3}{2}$ - channel
For the $|\pi^- p\rangle$ scattering we have $I = \frac{3}{2}$ and $I = \frac{1}{2}$.
This explains the additional structure.
The relative weight is given by the interaction strength for the different channels and the coupling of the original states to these different channels (Glebsch-Gordan coefficients)

So the relative weight for $|\pi^+ p\rangle$ becomes $|A_{3/2}|^2$

For $|\pi^- p\rangle$: $\begin{cases} \frac{1}{3}|A_{3/2}|^2 & \text{for } I = 3/2 \text{ - channel} \\ \frac{2}{3}|A_{1/2}|^2 & \text{for } I = 1/2 \text{ - channel.} \end{cases}$

6 + 2

11. (6 pt.) Mass relation in U-spin multiplet.
The mass M of a particular U-spin state $|U, U_3>$ (M = $<U, U_3|H|U, U_3>$) may be considered as a constant term m_0 equal for all members of a multiplet, a term m_s equal for all U-spin members with the same U, and a term m_v proportional to U_3; then M = $m_0 + m_s + m_v$; predict the mass relation for the neutral members of the $J^P=\frac{1}{2}^+$ baryon octet (see appendix B). The U-spin zero state is given by $\frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda^0)$.

We are considering a u-spin triplet: $|1,1\rangle = |n\rangle$; $|1,0\rangle = \frac{1}{2}(\sqrt{3}\,\Sigma^0 - \Lambda^0)$; $|1,-1\rangle = |\Xi^0\rangle$

must be orthogonal to $|0,0\rangle$, therefore it looks like this. !

because $\langle 1,0|0,0\rangle = \frac{1}{4}(\sqrt{3}\Sigma^0 - \Lambda^0)^*(\Sigma^0 + \sqrt{3}\Lambda^0)$
$= \frac{1}{4}(\sqrt{3}\underbrace{|\Sigma^0|^2}_{=1} - \sqrt{3}\underbrace{|\Lambda^0|^2}_{=1}) = 0.$

$U_3=+1: \langle 1,1|H|1,1\rangle = m_0 + m_s + m_v = m_n$

$U_3=0: \langle 1,0|H|1,0\rangle = m_0 + m_s = \frac{3}{4}m_{\Sigma^0} + \frac{1}{4}m_{\Lambda^0}$

$U_3=-1: \langle 1,-1|H|1,-1\rangle = m_0 + m_s - m_v = m_{\Xi^0}$

It follows that $\frac{1}{2}(m_n + m_{\Xi^0}) = \frac{3}{4}m_{\Sigma^0} + \frac{1}{4}m_{\Lambda^0}$ and there we have our mass-relation.

7

12. (7pt.) U-spin conservation.
The negatively charged $\frac{3}{2}^+$ baryons form a U-spin quartet, the neutral $\frac{1}{2}^+$ baryons form a U-spin triplet, and the negatively charged 0^- mesons form a U-spin doublet. Assuming U-spin conservation, determine the ratio of the decay-amplitudes for
$\Delta^- \to n\pi^-$ and $\Sigma^-(1385) \to nK^-$. (see appendix B and C).

$|\Delta^-\rangle = |\frac{3}{2}, \frac{3}{2}\rangle$

$|n\pi^-\rangle = |1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle = |\frac{3}{2},\frac{3}{2}\rangle$

~~[illegible]~~ strength $\Delta^- \to n\pi^-$: $|A_{3/2}|^2$

$|\Sigma^-\rangle = |\frac{3}{2},\frac{1}{2}\rangle$

$|nK^-\rangle = |1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\,|\frac{3}{2},\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}\,|\frac{1}{2},\frac{1}{2}\rangle$

strength $\Sigma^- \to nK^-$:
$|\langle \frac{3}{2},\frac{1}{2}|H|nK^-\rangle|^2 = \frac{1}{3}|A_{3/2}|^2$

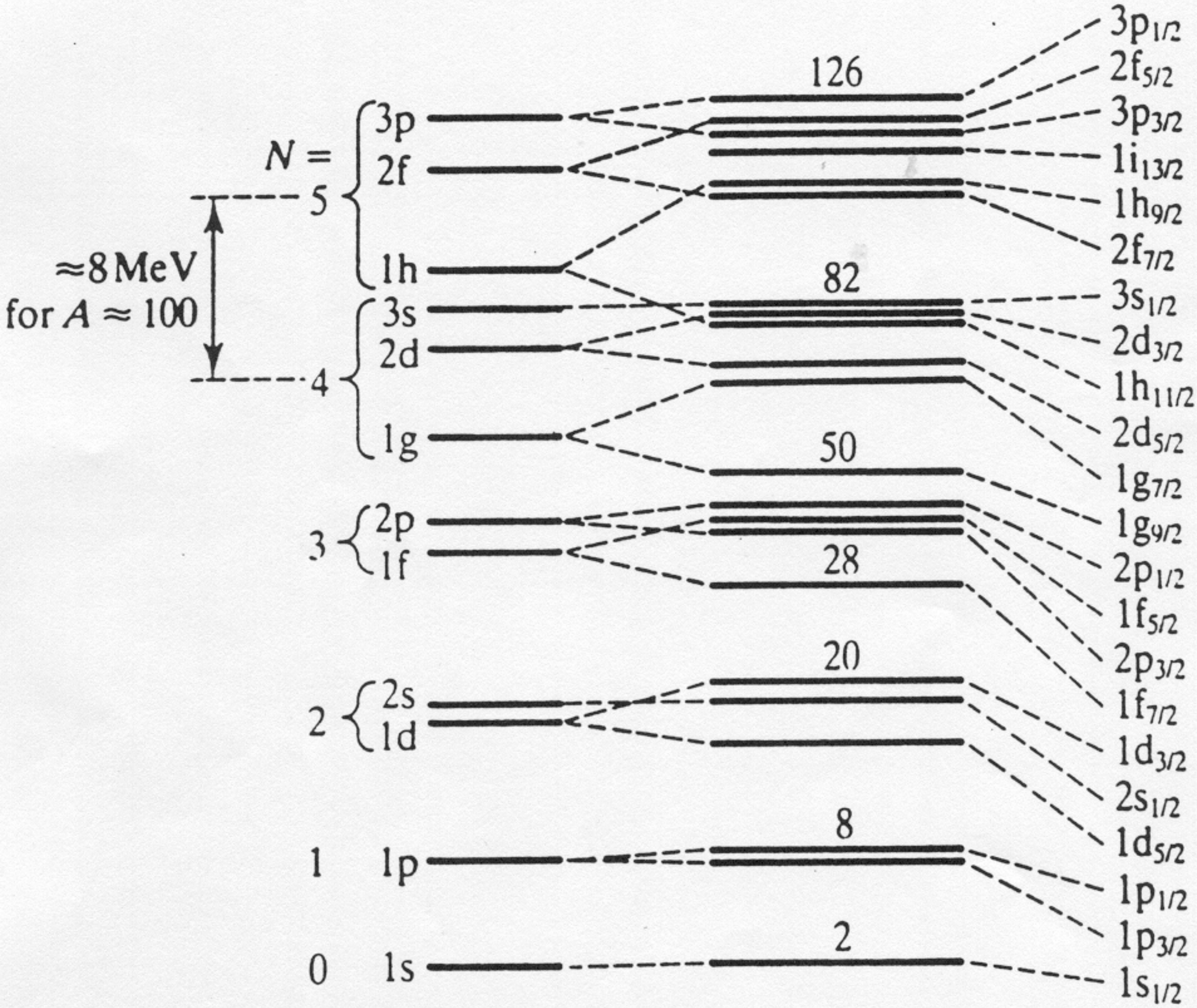
Two channels: $U=\frac{3}{2}$ and $U=\frac{1}{2}$, amplitudes $A_{3/2}$ and $A_{1/2}$.

$$\Rightarrow \frac{\text{amplitude}(\Delta^- \to n\pi^-)}{\text{amplitude}(\Sigma^- \to nK^-)} = \frac{1}{1/3} = 3.$$

Totaal te behalen punten: 75

Behaalde punten: 66,5
2 extra
68,5

Cijfer: 9,5


126
82
50
28
20
8
2
N =
5
4
3
2
1
0
≈8 MeV
for A ≈ 100
3p
2f
1h
3s
2d
1g
2p
1f
2s
1d
1p
1s
3p1/2
2f5/2
3p3/2
1i13/2
1h9/2
2f7/2
3s1/2
2d3/2
1h11/2
2d5/2
1g7/2
1g9/2
2p1/2
1f5/2
2p3/2
1f7/2
1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2

Appendix B

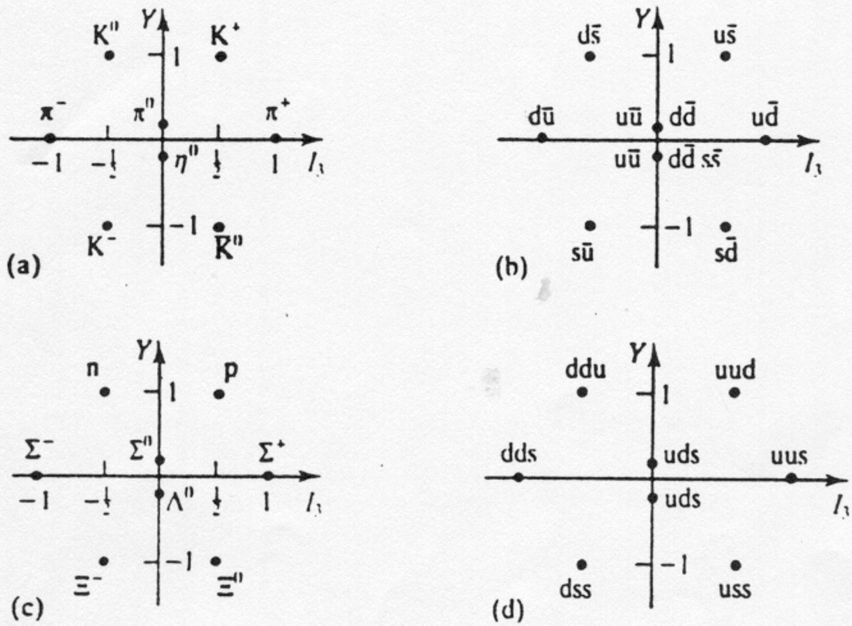


Figure 7.4
(a) The octet of 0^- mesons; (b) quark flavour assignments for the 0^- mesons; (c) the octet of $\frac{1}{2}^+$ baryons; (d) quark flavour assignments for the $\frac{1}{2}^+$ baryons.

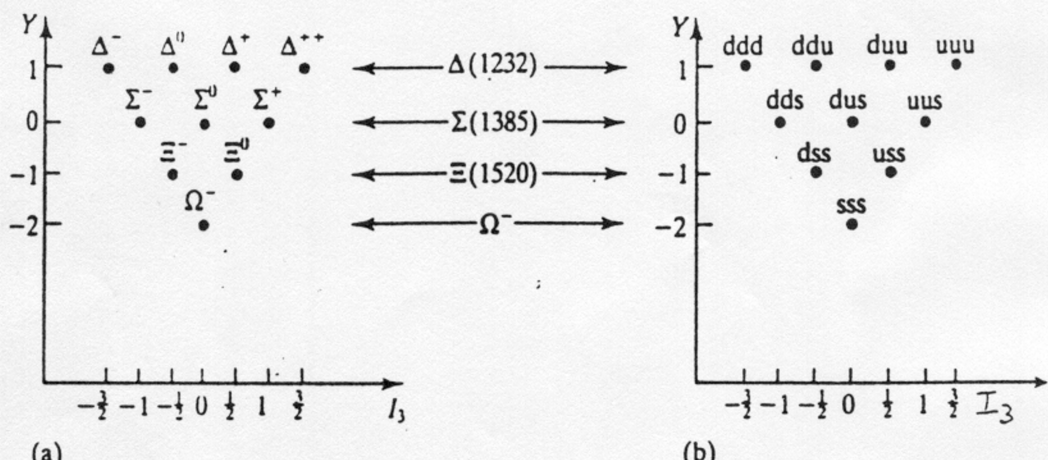


Figure 7.6
(a) The $\frac{3}{2}^+$ baryon decuplet and (b) its quark flavour content.